Apps of Derivatives Test Review

1. a) $s(t) = -16t^2 + 96t + 256$

$v(t) = -32t + 96$

$a(t) = -32$

b) $0 = -32t + 96$

$-96 = -32t$

$t = 3$ sec

c) $s(3) = -16(3)^2 + 96(3) + 256$

$s(3) = 400$ ft.

d) $0 = -16(t^2 - 6t - 16)$

$0 = -16(t-8)(t+2)$

$t = 8$ sec $\quad t = \cancel{-2}$ sec

e) $v(8) = -32(8) + 96$

$v(8) = -160$ ft/sec

f) $v(t)$: $+$ on $(0, 3)$, $-$ on $(3, 5)$; marks at 0, 3, 5

$s(0) = 256$ ft.

$s(3) = 400$ ft.

$s(5) = 336$ ft.

$|400 - 256| + |336 - 400|$

ft.

g) $s(5) - s(0)$

$336 - 256 = 80$ ft.

h) $144 = -16t^2 + 96t + 256$

$0 = -16t^2 + 96t + 112$

$0 = -16(t^2 - 6t - 7)$

$0 = -16(t-7)(t+1)$

$t = 7$ sec $\quad t = \cancel{-1}$ sec

$v(7) = -32(7) + 96$

$v(7) = -128$ ft/sec

2. 

$\frac{dV}{dt} = 500 \text{ cm}^3/\text{min}$

$\frac{dr}{dt} = ?$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

a) $r = 30$ cm

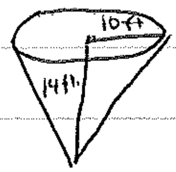
$(500) = 4\pi(30)^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{5}{36\pi}$ cm/min

b) $r = 60$ cm

$(500) = 4\pi(60)^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{5}{144\pi}$ cm/min

3. 

$\frac{dV}{dt} = -5 \text{ ft}^3/\text{sec}$

$\frac{dh}{dt} = ?$

$h = 7$ ft.

$\frac{r}{h} = \frac{10}{14}$

$14r = 10h$

$r = \frac{5h}{7}$

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{5h}{7}\right)^2 h$

$V = \frac{\pi}{3} \cdot \frac{25h^2}{49} \cdot h$

$V = \frac{25\pi}{147} h^3$

$\frac{dV}{dt} = \frac{75\pi}{147} h^2 \frac{dh}{dt}$

$(-5) = \frac{75\pi}{147}(7)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = -.064$ ft/sec

4. $f(x) = x^3 - 3x^2 - 24x + 2$ on $[-4, 8]$

$f'(x) = 3x^2 - 6x - 24$

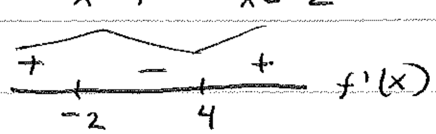
$0 = 3(x^2 - 2x - 8)$

$0 = 3(x-4)(x+2)$

$x = 4 \quad x = -2$

increasing: $(-\infty, -2) \cup (4, \infty)$

decreasing: $(-2, 4)$



relative max $(-2, 30)$

relative min $(4, -78)$

$f(-4) = -14$

$f(8) = 130$

absolute max: $(8, 130)$

absolute min: $(4, -78)$

$f''(x) = 6x - 6$

$0 = 6(x-1)$

$x = 1$

concave up: $(1, \infty)$

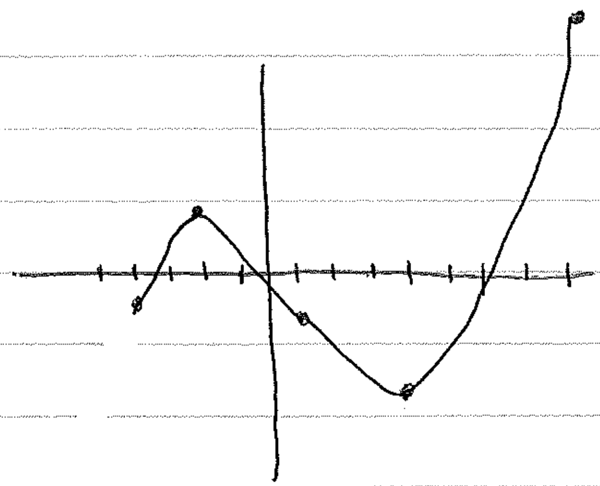
concave down: $(-\infty, 1)$

$-$ $\quad$ $+$ $\quad f''(x)$

1

poi

$(1, -24)$



5. $f(x) = (x+2)^{2/3}$ on $[-3, 6]$

$f'(x) = \frac{2}{3}(x+2)^{-1/3}$

$0 = \frac{2}{3(x+2)^{1/3}}$

increasing: $(-2, \infty)$

CN: $x = -2$

decreasing: $(-\infty, -2)$

$-$ $\quad$ $+$ $\quad f'(x)$

-2

relative min

$(-2, 0)$

$f(-3) = 1$

$f(6) = 4$

absolute max: $(6, 4)$

absolute min: $(-2, 0)$

$f''(x) = \frac{-2}{9}(x+2)^{-4/3}$

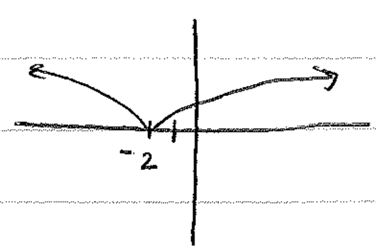
$0 = \frac{-2}{9(x+2)^{4/3}}$

concave up: none

concave down: $(-\infty, -2) \cup (-2, \infty)$

$-$ $\quad$ $-$ $\quad f''(x)$

-2



6. $f(x) = e^{x/2}$ on $[-1, 4]$

$f'(x) = e^{x/2} \cdot (\frac{1}{2})$

$f'(x) = \frac{e^{x/2}}{2}$

$f(-1) = \frac{1}{e^{1/2}}$

$f(4) = e^2$

$m = \frac{e^2 - \frac{1}{e^{1/2}}}{4+1} = \frac{e^{5/2} - 1}{5e^{1/2}}$

$\frac{e^{x/2}}{2} = \frac{e^{5/2} - 1}{5e^{1/2}}$

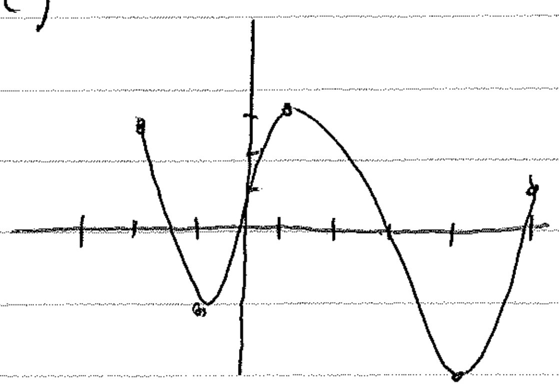
$x = 1.996$

7. a) relative max(s): $x = 1$ b/c $f'(x)$ changes from positive to negative

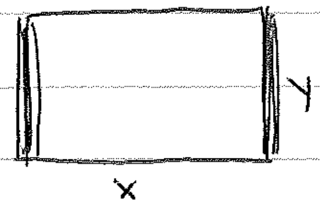
relative min(s): $x = -1$ & $x = 4$ b/c $f'(x)$ changes from negative to positive

b) point(s) of inflection: $x = 0$ & $x = 3$ b/c at $x = 0$ $f''(x)$ changes from pos to neg. and at $x = 3$ $f''(x)$ changes from neg. to pos.

c)



8.



$15000 = 4(2y) + 6(2x)$

$15000 = 8y + 12x$

$12x = 15000 - 8y$

$x = \frac{15000 - 8y}{12}$

$x = \frac{15000 - 8(937.5)}{12}$

$x = 625$ ft.

$A_{max} = xy$

$A_{max} = \left(\frac{15000 - 8y}{12}\right)y$

$A_{max} = \frac{15000}{12}y - \frac{2}{3}y^2$

$A_{max} = 12\ 0y - \frac{2}{3}y^2$

$A' = 1250 - \frac{4}{3}y$

$0 = 1250 - \frac{4}{3}y$

$\frac{4}{3}y = 1250$

$y = 937.5$ ft.

dimensions:

625 ft x 937.5 ft.

9. $s(t) = t^3 - 10t^2 + 27t - 18$

a) $v(t) = 3t^2 - 20t + 27$

$a(t) = 6t - 20$

b) $v(t) = 0$

$t = 1.880$ sec $\quad t = 4.786$ sec

c) $s(1.880) = 4.061$ m

$s(4.786) = -8.209$ m

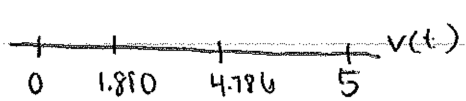
d) $a(t) = -8$

$t = 2$ sec

$v(2) = -1$ m/s $\quad s(2) = 4$ m

e) skip

f)



$s(0) = -18 \quad s(1.880) = 4.061$

$s(4.7863) = -8.209 \quad s(5) = -8$

$|4.061 + 18| + |-8.209 - 4.061| + |-8 + 8.209|$

34.539 m

g) $s(5) - s(0) = 10$ m

h) $s(8) = 70$

$|4.061 + 18| + |-8.209 - 4.061| + |70 + 8.209|$

112.539 m

i) $s(8) - s(0) = 88$ m

j) right: $[0, 1.880) \cup (4.7863, \infty)$ b/c $v(t) > 0$

left: $(1.880, 4.786)$ b/c $v(t) < 0$

k) $\frac{s(9)-s(0)}{9-0} = \frac{144+18}{9} = 18$ m/s

l) $\frac{s(7)-s(3)}{7-3} = \frac{24-0}{4} = 6$ m/s

10. a) increasing: $(-3,0) \cup (2,3)$ b/c $f'(x)>0$

decreasing: $[-4,-3) \cup (0,2)$ b/c $f'(x)<0$

b) relative max(s): $x=0$ b/c $f'(x)$ changes from pos to neg

relative min(s): $x=-3$ & $x=-2$ b/c $f'(x)$ change from neg to pos

c) concave up: $[-4,-1.5) \cup (1,3]$ b/c $f''(x)>0$

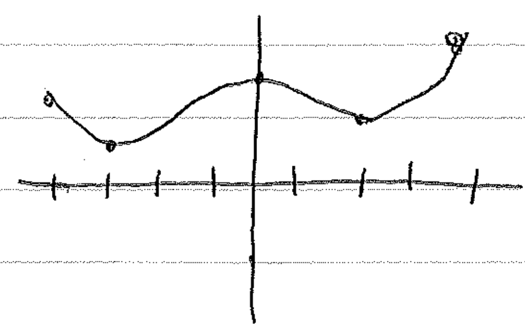
concave down: $(-1.5,1)$ b/c $f''(x)<0$

d) point(s) of inflection

$x=-1.5$ b/c $f'(x)$ changes from increasing to decreasing

$x=1$ b/c $f'(x)$ changes from decreasing to increasing

e)



11. $s(t)=t^2-5t-\sin(2t)+2$

a) $v(t)=2t-5-2\cos(2t)$

$a(t)=2+4\sin(2t)$

b) $v(t)=0$

$t=1.508, 2.208, 3.384$ sec

c) $s(1.508)=-3.391$ ft.

$s(2.208)=-3.208$ ft.

$s(3.384)=-3.935$ ft.

d) $v(t) = 3$

$t = 3.951$ sec

$s(3.951) = -3.1425$ ft.

e) $\underset{1.508}{-} \quad + \quad \underset{2.208}{-} \quad \underset{3.384}{+} \quad v(t)$

right: $(1.508, 2.208) \cup (3.384, \infty)$ b/c $v(t) > 0$

left: $[0, 1.508) \cup (2.208, 3.384)$ b/c $v(t) < 0$

f) $s(7) - s(0) = 13.009$ ft.

g) $s(0) = 2$

$s(1.508) = -3.391$

$s(2.208) = -3.935$

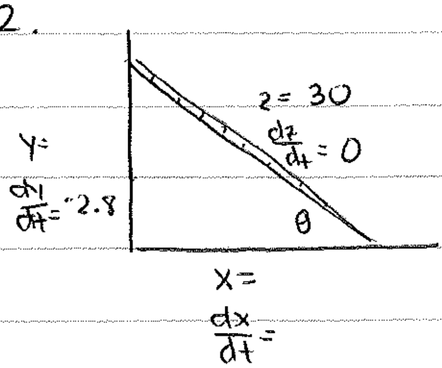
$s(7) = 15.009$

$|-3.391 - 2| + |-3.935 + 3.391| + |15.009 + 3.935|$

25.244 ft.

h) $\dfrac{s(8) - s(0)}{8 - 0} = \dfrac{26.288 - 2}{8} = 3.036$ ft/sec

12.



a) $x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$(29.732)\frac{dx}{dt} + (4)(-2.8) = (30)(0)$

$\frac{dx}{dt} = .377$ ft/sec

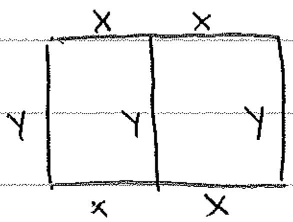
b) $\sin\theta = \frac{y}{z}$

$\cos\theta \frac{d\theta}{dt} = \dfrac{\frac{dy}{dt}z - y\frac{dz}{dt}}{z^2}$

$\left(\frac{15}{30}\right)\frac{d\theta}{dt} = \dfrac{(-2.8)(30) - 0}{(30)^2}$

$\frac{d\theta}{dt} = -.187$ rad/sec

13.

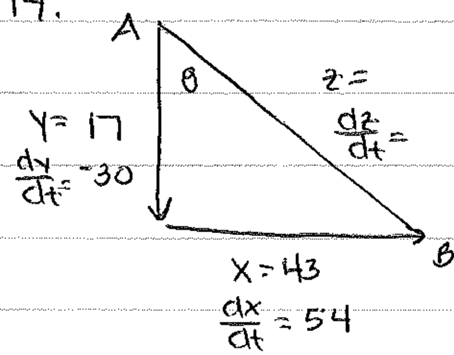


$$C_{min} = 3(4x) + 3(2y) + 2y$$
$$C_{min} = 12x + 8y$$
$$C_{min} = 12x + 8\left(\frac{5400}{x}\right)$$
$$C_{min} = 12x + 43200x^{-1}$$
$$C' = 12 - 43200x^{-2}$$
$$0 = 12 - \frac{43200}{x^2}$$
$$\frac{43200}{x^2} = 12$$
$$x = 60 \text{ ft.}$$

$$10{,}800 = 2xy$$
$$y = \frac{5400}{x}$$
$$y = \frac{5400}{(60)}$$
$$y = 90 \text{ ft.}$$

each pen: 60 ft × 90 ft.

14.



a) $x^2 + y^2 = z^2$

$$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}z\frac{dz}{dt}$$
$$(17)(-30) + (43)(54) = (46.239)\frac{dz}{dt}$$
$$\frac{dz}{dt} = 39.189 \text{ mph}$$

b) $\sin\theta = \frac{x}{z}$

$$\cos\theta\frac{d\theta}{dt} = \frac{\frac{dx}{dt}(z) - (x)\frac{dz}{dt}}{(z)^2}$$

$$\left(\frac{17}{46.239}\right)\frac{d\theta}{dt} = \frac{(54)(46.239) - (43)(39.189)}{(46.239)^2}$$

$$\frac{d\theta}{dt} = 1.033 \text{ rad/hr}$$

15. a) $s(t) = -16t^2 + 276t + 73$

$v(t) = -32t + 276$

$a(t) = -32$

b) $v(t) = 0$

$t = 8.625$ sec

$s(8.625) = 1263.25$ ft.

c) $s(t) = 100$

$t = .098,\ 17.152$ sec

$v(.098) = 272.851$ ft/sec

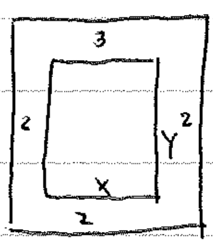
$v(17.152) = -272.851$ ft/sec

d) $s(t) = 0$

$t = 17.512$ sec

$v(17.512) = -284.339$ ft/sec

16.



$xy = 2400$

$y = \frac{2400}{x}$

$y = \frac{2400}{(43.818)}$

$y = 54.772$ ft.

$A_{min} = (x+4)(y+5)$

$A_{min} = xy + 5x + 4y + 20$

$A_{min} = \cancel{x}\left(\frac{2400}{\cancel{x}}\right) + 5x + 4\left(\frac{2400}{x}\right) + 20$

$A_{min} = 5x + 9600x^{-1} + 20$

$A' = 5 - 9600x^{-2}$

$0 = 5 - \frac{9600}{x^2}$

$\frac{9600}{x^2} = 5$

$x = 43.818$ ft.

pool dimensions: 43.818 ft $\times$ 54.772 ft.